Supplemental Document

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Single pixel polarimetric imaging through scattering media: supplement

KAI LING C. SEOW,¹ PETER TÖRÖK,² AND MATTHEW R. FOREMAN^{1,*}

¹Blackett Laboratory, Department of Physics, Imperial College London, Prince Consort Road, London SW7 2AZ, UK

²Division of Physics and Applied Physics, Nanyang Technological University, Singapore, Singapore *Corresponding author: matthew.foreman@imperial.ac.uk

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Single Pixel Polarimetric Imaging through Scattering Media: Supplemental Document

This document provides supplementary information for "Single Pixel Polarimetric Imaging through Scattering Media". Some theoretical background and experimental data for the full Mueller matrix images obtained with and without a scattering medium present using single pixel polarimetry imaging are presented.

1. CONSTRAINED LEAST SQUARES ALGORITHM

Estimating the true Mueller matrix **M** of a sample, by direct inversion of the instrument matrices (**A** and **W**) defined in the main text, i.e. by using the equation

$$\hat{\mathbf{M}} = \mathbf{A}^{-1} \mathbf{D} \mathbf{W}^{-1},\tag{S1}$$

where **D** is a matrix of intensities measured by a polarisation state analyser, is not guaranteed to provide a physically acceptable estimate $\hat{\mathbf{M}}$. In this work, we instead use a constrained least squares algorithm to ensure physicality of Mueller matrices computed from experimental data, similar to the maximum likelihood-based algorithm proposed by Aiello *et al.* [1]. Specifically, the constrained least squares algorithm seeks to find the Mueller matrix, $\hat{\mathbf{M}}$, that minimises the function

$$\mathcal{F}(\hat{\mathbf{M}}) = \|\mathbf{D} - \mathbf{A}\hat{\mathbf{M}}\mathbf{W}\|_{2},\tag{S2}$$

i.e. $\hat{\mathbf{M}}$ minimises the difference between the theoretical and experimental intensities and is the estimate of the true Mueller matrix, \mathbf{M} . To enforce physicality of the Mueller matrix we make use of the related \mathbf{H} matrix [2], which can be obtained from $\hat{\mathbf{M}}$ as

$$\mathbf{H} = \frac{1}{4} \sum_{k,l=0}^{3} m_{kl} (\boldsymbol{\sigma}_k \otimes \boldsymbol{\sigma}_l^*),$$
(S3)

where m_{kl} denotes the (k, l) th element of $\hat{\mathbf{M}}$, and σ_k are the 2 × 2 Pauli matrices. It can be shown that the elements of \mathbf{H} can be expressed in terms of the ensemble averaged products of elements T_{ij} of an associated Jones matrix, i.e. elements of \mathbf{H} are of the form $\langle T_{ij}T_{kl} \rangle$ $(i, j, k, l \in (0, 1))$. More specifically, \mathbf{H} is the complex correlation matrix of the underlying Jones matrix elements [1]. Consequently, for a Mueller matrix to be physically acceptable, its associated \mathbf{H} matrix must be positive semi-definite [2]. Any positive semi-definite matrix can be represented using the Cholesky decomposition whereby $\mathbf{H} = \mathbf{LL}^{\dagger}$ where \mathbf{L} is a lower triangular matrix containing 16 real parameters of the form [1]

$$\mathbf{L} = \begin{bmatrix} l_1 & 0 & 0 & 0\\ l_5 + il_6 & l_2 & 0 & 0\\ l_{11} + il_{12} & l_7 + il_8 & l_3 & 0\\ l_{15} + il_{16} & l_{13} + il_{14} & l_9 + il_{10} & l_4 \end{bmatrix}.$$
 (S4)

Using Eq. (S2), Eq. (S3) and Eq. (S4), an optimal set of parameters $\{l_1, l_2, \ldots, l_{16}\}$ that minimises \mathcal{F} can be found through a minimisation algorithm such as the fminsearch function found in MATLAB. The corresponding optimised **H** matrix, \mathbf{H}_{opt} , can then be reconstructed using $\mathbf{H} = \mathbf{LL}^{\dagger}$, and the final estimated Mueller matrix, now guaranteed to be physical, can be computed elementwise using

$$\hat{m}_{kl} = \operatorname{tr}[\mathbf{H}_{opt}[\boldsymbol{\sigma}_k \otimes \boldsymbol{\sigma}_l^*]]. \tag{S5}$$

2. VALIDITY OF NEGLECTING B_{ml}

To justify the assumption that $\mathbf{B}_{ml} \ll \mathbf{A}_m$ used in the main text we here present results from Monte Carlo simulations. For computational reasons we limit our computations to a 2D geometry, however, similar results are expected for 3D simulations.

The matrix \mathbf{B}_{ml} was defined in the main text as $\mathbf{B}_{ml} = \sum_{n} (\mathbf{T}_{nm}^{SM} \otimes \mathbf{T}_{nl}^{SM,*})$. Explicitly, this can be written as

$$\mathbf{B}_{ml} = \sum_{n} \begin{bmatrix} T_{nm,00} T_{nl,00}^{*} & T_{nm,00} T_{nl,01}^{*} & T_{nm,01} T_{nl,00}^{*} & T_{nm,01} T_{nl,01}^{*} \\ T_{nm,00} T_{nl,10}^{*} & T_{nm,00} T_{nl,11}^{*} & T_{nm,01} T_{nl,10}^{*} & T_{nm,01} T_{nl,11}^{*} \\ T_{nm,10} T_{nl,00}^{*} & T_{nm,10} T_{nl,01}^{*} & T_{nm,11} T_{nl,00}^{*} & T_{nm,11} T_{nl,01}^{*} \\ T_{nm,10} T_{nl,10}^{*} & T_{nm,10} T_{nl,11}^{*} & T_{nm,11} T_{nl,10}^{*} & T_{nm,11} T_{nl,11}^{*} \end{bmatrix},$$
(S6)

where $T_{nm,pq}$ is the $(p,q)^{th}$ element in \mathbf{T}_{nm}^{SM} , and the superscript, 'SM', has been omitted for clarity. The matrix \mathbf{A}_m has the same form as \mathbf{B}_{ml} , except that *l* is replaced by *m* (i.e. $\mathbf{A}_m = \mathbf{B}_{mm}$). As such, the Jones matrix \mathbf{T}_{nm}^{SM} is required to compute \mathbf{A}_m and \mathbf{B}_{ml} . To calculate \mathbf{T}_{nm}^{SM} numerical simulations were used (see Ref. [3] for full details). Simulations

To calculate T_{nm}^{SM} numerical simulations were used (see Ref. [3] for full details). Simulations considered light (wavelength $\lambda_0 = 638$ nm) incident on a scattering medium (of transverse width W = 1 mm) composed of non-overlapping uniformly randomly distributed cylinders (refractive index of 1.6 and radius 220 nm) in air (refractive index of 1). Note, these parameters were chosen such that the simulated scattering medium had a mean free path and scattering anisotropy factor comparable to real biological samples. Cylinders were assumed to be oriented with their axis perpendicular to the incident light and to be infinitely long. Since the radius of the cylinders was taken to be much smaller than the wavelength of light, the cylinders were approximated as line dipoles. The effective polarisability per unit length of the cylinders [4]. A coupled dipole formalism [5] was then used to calculate the resulting far field for a given input field. In order to conduct the numerical study in a manner that was independent of the design of the collection optics, the resulting far-field was calculated on a cylindrical detector with a radius of 10000 λ_0 .

The Jones matrix of the scattering medium at the n*th* output pixel from an incident field at the m*th* input pixel is defined as

$$\mathbf{T}_{nm}^{SM} = \begin{bmatrix} T_{nm,00} & T_{nm,01} \\ T_{nm,10} & T_{nm,11} \end{bmatrix},$$
(S7)

such that

$$\begin{bmatrix} E_{n,\parallel} \\ E_{n,\perp} \end{bmatrix} = \begin{bmatrix} T_{nm,00} & T_{nm,01} \\ T_{nm,10} & T_{nm,11} \end{bmatrix} \begin{bmatrix} E_{m,\parallel} \\ E_{m,\perp} \end{bmatrix} , \qquad (S8)$$

where $(E_{m\perp}, E_{m\parallel})$ are the two orthogonal components of the input field at the *m*th pixel, and $(E_{n\perp}, E_{n\parallel})$ are similarly defined for the *n*th output pixel. Note that the perpendicular and parallel components of the scattered field, $E_{n,\perp}$ and $E_{m,\parallel}$, are defined with reference to the plane spanned by the cylinder axis and the scattering direction. From Equation S8, it can be seen that the Jones matrix \mathbf{T}_{nm}^{SM} can be found by separately calculating the fields scattered for the incident fields, $\vec{E}_{\parallel} = [E_{m,\parallel}, 0]^{\top}$ and $\vec{E}_{\perp} = [0, E_{m,\perp}]^{\top}$. Using Equation S6, \mathbf{A}_m and \mathbf{B}_{ml} can then be computed from the simulated Jones matrix.

To test the hypothesis put forward in the main text, that is, that $\mathbf{B}_{ml} \ll \mathbf{A}_m$ for pixel sizes much larger than a length scale κ which is determined by the smallest of the translation correlation length [6] or the average speckle size [3], the scattered far fields from two adjacent input pixels were found in separate simulations. The pixel widths were chosen to be 200 microns, such that the output speckle field was uncorrelated. In total 200 realisations of disorder were simulated. For each instance of disorder, \mathbf{A}_m and \mathbf{B}_{ml} were computed and normalised by the total intensity transmission from the m*th* pixel across the two input polarisation states. A histogram of the magnitude of the elements of both \mathbf{A}_m and \mathbf{B}_{ml} is shown in Figures S1(a)-(c) for scattering media of three different thicknesses, namely $R = L/l_{tr} = 1, 2, 3$ where l_{tr} is the transport mean free path for incident light polarised perpendicular to the cylinder axis. In all three cases, the values for \mathbf{A}_m are seen to be much greater than those of \mathbf{B}_{ml} . Due to the cylindrical geometry used in these simulations, the Jones matrix is inherently diagonal with off-axis elements that are equal to zero



Fig. S1. A_m (red bars) versus B_{ml} (blue bars) for (a) R = 1, (b) R = 2 and (c) R = 3. All matrix elements, other than those labelled, lie within the blue part of the histogram. (d) R = 3 in the presence of significant aberrations.

[4], so elements in \mathbf{A}_m and \mathbf{B}_{ml} with contributions from these off-axis elements would also be equal to zero. As such, only four of the elements in \mathbf{A}_m and \mathbf{B}_{ml} are non-zero.

As discussed further in Ref. [3], similar results can be expected for input pixel sizes larger than the width of the correlation function, κ , of the scattering medium. When the pixel size is smaller than κ the output speckle from two adjacent input pixels is correlated such that the correlations described by the elements of \mathbf{B}_{ml} are not negligible. Furthermore, the magnitude of elements of \mathbf{B}_{ml} may also be comparable to \mathbf{A}_m when the field incident on the scattering medium from adjacent input pixels overlaps significantly. In single pixel polarimetric imaging, the spatial masks on the digital micromirror device (DMD) are imaged onto the test object which is located right next to the scattering medium. Significant aberrations in the imaging optics can hence cause a spreading of the point spread function, in turn causing the imaged DMD pixels to partially overlap. Figure S1(d) shows the elements in \mathbf{A}_m and \mathbf{B}_{ml} when there is a partial overlap between the pixels illuminating the scattering medium. It can be seen that in such a scenario, the elements in \mathbf{B}_{ml} can no longer be considered to be negligible compared to the elements in \mathbf{A}_m . As such, care should be taken to minimise the aberrations in the design of a single pixel polarimetric imaging system.

3. EXPERIMENTAL MUELLER MATRIX IMAGES



Fig. S2. Spatially resolved Mueller matrix for the test object without any scattering medium present, with pixels in all matrix elements other than the M_{00} element normalised to their respective M_{00} values.



Fig. S3. Spatially resolved Mueller matrix for the test object with SM1 present, with pixels in all matrix elements other than the M_{00} element normalised to their respective M_{00} values.



Fig. S4. Spatially resolved Mueller matrix for the test object with SM2 present, with pixels in all matrix elements other than the M_{00} element normalised to their respective M_{00} values.



Fig. S5. Spatially resolved Mueller matrix for the test object with SM3 present, with pixels in all matrix elements other than the M_{00} element normalised to their respective M_{00} values.

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